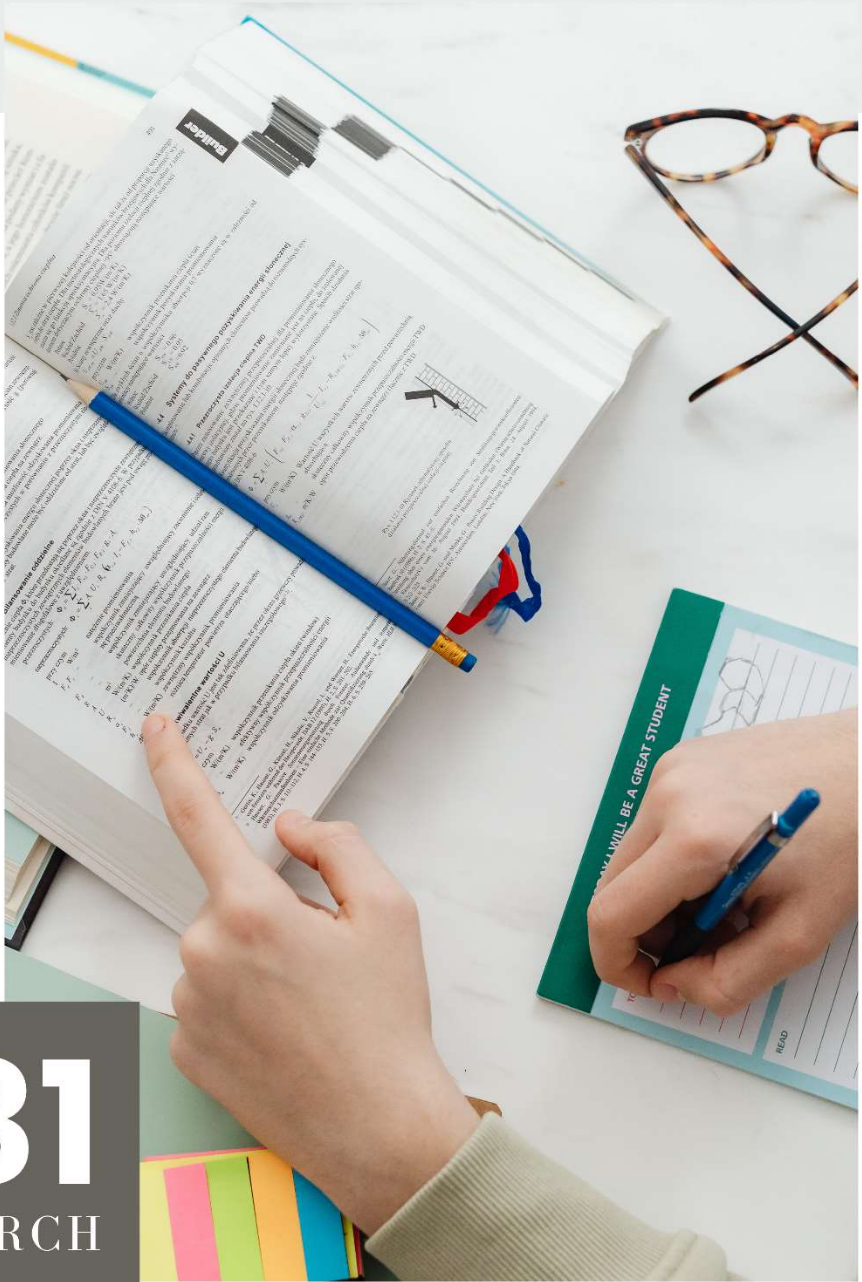
III INTERNATIONAL SCIENTIFIC AND PRACTICAL CONFERENCE 2026

COLLECTION

Innovation, integration and modern problems in the scientific activities of young researchers and students: theory and practice



31 MARCH

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Jizzakh, Uzbekistan

MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATION OF THE REPUBLIC OF UZBEKISTAN

JIZZAKH BRANCH OF THE NATIONAL UNIVERSITY OF UZBEKISTAN NAMED AFTER MIRZO ULUGBEK

SCIENTIFIC JOURNAL OF SCIENCE TECHNOLOGY & DIGITAL FINANCE

JOURNAL OF INTERNATIONAL SCIENCE NETWORKS

Innovation, integration and modern problems in the scientific activities of young researchers and students: theory and practice collection of materials of the international scientific and practical conference on the topic

(March 31, 2026)

Jizzakh-2026

Innovation, integration and modern problems in the scientific activities of young researchers and students: theory and practice – Jizzakh: Department of economics and tourism of Jizzakh branch of the national university of Uzbekistan named after Mirzo Ulugbek, March 31, 2026, 990 pp.

Editors in charge: Ass.prof. Soy M.P.

In the collection of materials of the conference, the role and role of Science, Education and production in the era of globalization, the pressing problems of the issues of interaction of these processes, feedback on their solutions were presented by mature specialists of the field.

In addition, research on the scientific and practical topic, carried out in the economics, Exact Sciences, Natural Sciences and socio-humanities during the globalization period, information is presented in the scientific and practical fields, which includes the latest innovative technologies in the fields of production.

It can be argued that this collection is one of the specific intersections of current thoughts and innovative ideas of the world of science. This scientific and practical conference was actively attended by professors and scientific researchers engaged in scientific research in Uzbekistan and foreign countries. In increasing the position of the scientific and practical conference, the professors and teachers of domestic and foreign higher educational institutions made a significant contribution.

Professors and teachers of foreign higher educational institutions who actively participated in the work of the conference made a worthy contribution to the high level of interaction with scientists of our country. The processes of international cooperation with foreign countries and exchange with them in the field of Science in the era of globalization have a positive effect on the development of Higher Education, the fields of Science and production. The materials of this conference are special in that they include a wide range of research, from theoretical developments to practical solutions, demonstrating the diversity of approaches and directions in this area.

In conclusion, it should be noted that this scientific and practical conference will be a very useful collection for everyone who is interested in modern research in the fields of further development of Higher Education, Science, Education and production in the era of globalization. The authors are responsible for the content and quality of the articles and abstracts included in the collection.

EUCLID'S THEOREM AND ITS APPLICATIONS IN NUMBER THEORY AND MODERN COMPUTATION

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Abstract: Classical number theory contains several foundational principles that continue to influence modern mathematics and computer science. Among these principles, the theorem associated with the ancient Greek mathematician Euclid provides a systematic method for analyzing divisibility and determining the greatest common divisor of integers. The theorem forms the theoretical basis of the Euclidean algorithm, one of the oldest and most efficient algorithms in mathematics. Over time, the ideas derived from Euclid's work have become essential in fields such as algebra, computational mathematics, and modern cryptography. The present study examines Euclid's theorem through a formal theorem–proof framework, demonstrating its logical structure and its applications in number theory, including Euclid's lemma and the proof of the infinitude of prime numbers. The discussion also highlights the relevance of these classical ideas in contemporary computational systems.

Keywords: Euclid's theorem, Euclidean algorithm, number theory, prime numbers, modular arithmetic, cryptographic algorithms

The foundations of number theory are closely connected with the systematic reasoning developed in ancient Greek mathematics. Among the earliest comprehensive treatments of mathematical logic is the work *Elements*, written by Euclid around 300 BCE. This treatise organized mathematical knowledge through axioms, definitions, and propositions, establishing a model of rigorous reasoning that remains influential today. Within this framework, Euclid introduced methods for studying divisibility properties of integers and demonstrated how complex numerical relationships could be reduced to simpler components through logical deduction. One of the most important results emerging from this tradition concerns the method for determining the greatest common divisor of two integers, which later became known as the Euclidean algorithm. The logical reasoning behind this method is often referred to as Euclid's theorem.

The fundamental principle underlying this theorem can be expressed formally through the following statement.

Theorem (Euclid's Theorem on the Greatest Common Divisor). Let a and b be positive integers with $a > b$. Then the greatest common divisor of $a$and b is equal to the greatest common divisor of b and the remainder obtained when a is divided by b.

Proof: Suppose that d is a common divisor of both a and b. Then d divides any linear combination of these numbers. In particular, since $a = qb + r$ for some integer q and remainder r, the number $r = a - qb$ must also be divisible by d. Therefore every common divisor of a and b is also a divisor of b and r. Conversely, if a number divides both b and r, then it must divide $a = qb + r$. Thus the sets of common divisors of the pairs (a, b) and (b, r) are identical, implying that their greatest common divisors are equal. ■

This theorem provides the logical basis for a recursive procedure known as the Euclidean algorithm. By repeatedly replacing the pair (a, b) with $(b, a \bmod b)$, the numbers involved become progressively smaller until the remainder becomes zero. The final non-zero remainder obtained through this process is the greatest common divisor of the original integers.

To illustrate the practical use of this theorem, consider the integers 252 and 105. Applying the Euclidean algorithm produces the following sequence of divisions:

$$252 = 105 \times 2 + 42$$
$$105 = 42 \times 2 + 21$$
$$42 = 21 \times 2 + 0$$

Since the final non-zero remainder is 21, it follows that the greatest common divisor of 252 and 105 is 21. This example demonstrates the remarkable efficiency of Euclid's reasoning, as even relatively large integers can be reduced to their fundamental divisibility relationships through a small number of steps.

The theoretical importance of Euclid's theorem becomes even clearer when examining its role in proving other fundamental results in number theory. One such result is known as Euclid's lemma, which describes an important property of prime numbers.

Theorem (Euclid's Lemma).
If a prime number p divides the product of two integers ab, then p divides a or p divides b.

Proof: Assume that p divides ab but does not divide a. Since p is prime, the greatest common divisor of p and a must be 1. According to Bézout's identity, there exist integers x and y such that

$$px + ay = 1.$$

Multiplying both sides by b gives

$$pbx + aby = b.$$

Because p divides both terms on the left-hand side, it must divide b. Therefore p divides a or p divides b. ■

This lemma forms the basis for one of the most elegant arguments in classical mathematics: Euclid's proof that there are infinitely many prime numbers. The reasoning proceeds by assuming that only finitely many primes exist and demonstrating that this assumption leads to a contradiction.

Theorem (Infinitude of Prime Numbers).
There exist infinitely many prime numbers.

Proof: Assume that the number of primes is finite and that they are $p_1, p_2, p_3, \dots, p_n$. Consider the number

$$N = p_1 p_2 p_3 \cdots p_n + 1$$

None of the primes in the list divides N, since dividing $N$by any $p_i$leaves a remainder of 1. Therefore either Nitself is prime or it contains a prime factor not included in the list. In either case, a new prime number exists, contradicting the assumption that the list contained all primes. Hence there must be infinitely many primes. ■

Although these results originated in ancient mathematical theory, their importance has grown considerably with the development of modern computing. The Euclidean algorithm remains one of the most efficient methods for determining greatest common divisors, with computational complexity proportional to the logarithm of the numbers involved. This efficiency makes the algorithm particularly valuable in cryptography, where large integers must be manipulated rapidly.

One of the most prominent applications occurs in public-key encryption systems such as the RSA cryptosystem. In such systems, it is often necessary to determine the multiplicative inverse of an integer in modular arithmetic. If two integers $a$and $n$are relatively prime, then there exists an integer $x$satisfying

$$ax \equiv 1 (\mathrm{mod} n)$$

The extended Euclidean algorithm provides a direct method for computing this inverse by determining coefficients that satisfy Bézout's identity. Without this algorithm, many cryptographic protocols would be computationally impractical.

The continuing relevance of Euclid's theorem illustrates the remarkable durability of classical mathematical ideas. What began as an investigation of simple divisibility relationships has evolved into a cornerstone of modern computational mathematics. The theorem not only provides a powerful algorithmic tool but also serves as a foundation for fundamental concepts in number theory, including prime numbers, modular arithmetic, and cryptographic security. Through these applications, Euclid's reasoning continues to influence both theoretical mathematics and modern technological systems.

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