MIRZO ULUG‘BEK NOMIDAGI
O‘ZBEKISTON MILLIY UNIVERSITETI
JIZZAX FILIALI

KOMPYUTER ILMLARI VA MUHANDISLIK TEXNOLOGIYALARI

XALQARO ILMIY-TEXNIK ANJUMAN MATERIALLARI TO‘PLAMI

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MIRZO ULUG‘BEK NOMIDAGI O‘ZBEKISTON MILLIY UNIVERSITETINING JIZZAX FILIALI

KOMPYUTER ILMLARI VA MUHANDISLIK TEXNOLOGIYALARI

mavzusidagi Xalqaro ilmiy-texnik anjuman materiallari to‘plami

(2025-yil 26-27-sentabr)

2-QISM

JIZZAX-2025

Kompyuter ilmlari va muhandislik texnologiyalari. Xalqaro ilmiy-texnik anjuman materiallari to‘plami – Jizzax: O‘zMU Jizzax filiali, 2025-yil 26-27-sentabr. 368-bet.

Xalqaro miqyosidagi ilmiy-texnik anjuman materiallarida zamonaviy kompyuter ilmlari va muhandislik texnologiyalari sohasidagi innovatsion tadqiqotlar aks etgan.

Globallashuv sharoitida davlatimizni yanada barqaror va jadal sur’atlar bilan rivojlantirish bo‘yicha amalga oshirilayotgan islohotlar samarasini yaxshilash sohasidagi ilmiy-tadqiqot ishlariga alohida e’tibor qaratilgan. Zero iqtisodiyotning, ijtimoiy sohalarni qamrab olgan modernizatsiya jarayonlari, hayotning barcha sohalarini liberallashtirishni talab qilmoqda.

Ushbu ilmiy ma’ruza tezislari to‘plamida mamlakatimiz va xorijlik turli yo‘nalishlarda faoliyat olib borayotgan mutaxassislar, olimlar, professor-o‘qituvchilar, ilmiy tadqiqot institutlari va markazlarining ilmiy xodimlari, tadqiqotchilari, magistr va talabalarning ilmiy-tadqiqot ishlari natijalari mujassamlashgan.

• comet3(x,y,z,p)- avvalgi komandaga o‘xshash, faqat kometa izining uzunligini ham ko‘rsatish mumkin. Kometaning izi p*length(y) ko‘rinishida beriladi (length(y)-y vektorning o‘lchami, p<1, sukut bo‘yicha p=0,1).

Umuman olganda MATLAB matematikaning rivojlanishi davomida to‘plangan matematik hisoblashlar bo‘yicha tajribani o‘zida mujassamlashtirgan va uni grafik vizuallash va animasiya vositalari bilan uyg’unlashtirilgan.

Foydalanilgan adabiyotlar ro‘yxati:

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USE DIAGONALIZATION TO RAISE O MATRIX TO A HIGH POWER

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Abstract. This thesis discusses the method of using diagonalization to raise a matrix to a high power. Diagonalization provides an efficient way to simplify complex computations, especially for large matrices. The process of powering a diagonal matrix and reconstructing the original matrix is explained. Practical examples are presented to demonstrate the advantages of the method.

Keywords. Matrix, diagonalization, eigenvalues, eigenvectors, matrix power, linear algebra.

Suppose we have a matrix and we want to find A^{50}. One could try to multiply A with itself 50 times, but this is a lot of work (try it!). However, diagonalization allows us to compute high powers of a matrix relatively easily. Suppose A is diagonalizable, so that $P^{-1}AP = D$. We can rearrange this equation to write $A = PDP^{-1}$.Now, consider A^2 . Since $A = PDP^{-1}$, it follows that

$$A^2 = (PDP^{-1})^2 = PDP^{-1}PDP^{-1} = PD^2P^{-1}.$$

Similarly,

$$A^3 = (PDP^{-1})^3 = PDP^{-1}PDP^{-1} = PD^2P^{-1}$$

In general,

$$\boldsymbol{A^n = (PDP^{-1})^n = PD^nP^{-1}}$$

$Therefore, we\ have\ reduced\ the\ problem\ to\ finding\ D^n. But\ as\ we\ saw\ in\ Example$ 8.1
wers of matrices.

Example 1: Raising a matrix to a high power

$$\text{Let } A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \quad . \, Find\ A^{50}.$$

Solution. First,we will diagonalize A.Following the usual steps, we find that the eigenvalues are $\lambda = 1$ and $\alpha = 2$. The basic eigenvectors corresponding to $\lambda = 1$ are

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

and the basic eigenvector corresponding to λ=2 is

$$v_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

Now we construct P by using the basic eigenvectors of A as the columns of P . Thus

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Then

$$P^{-1}AP = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = D.$$

Now follows by rearranging the equation that $A = PDP^{-1}$. And therefore,as noted above.

$A^{50} = PD^{50}P^{-1} =$

$$\begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1^{50} & 0 & 0 \\ 0 & 1^{50} & 0 \\ 0 & 0 & 2^{50} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2^{50} & -1+2^{50} & 0 \\ 0 & 1 & 0 \\ 1-2^{50} & 1-2^{50} & 1 \end{bmatrix}$$

Thus,trough diagonalization, we have efficiently computed a high power of A. The following example shows that we can also use the same technique for finding a square root of a matrix.

$$Let\ A = \begin{bmatrix} 1 & 3 & 3 \\ -1 & 5 & 3 \\ 1 & -1 & 1 \end{bmatrix} . Find\ a\ square\ root\ of\ a\ matrix\ B.$$

Solution.We first diagonalize A. The charactirestic polynomial is

$$\begin{bmatrix} 1-\lambda & 3 & 3 \\ -1 & 5-\lambda & 3 \\ 1 & -1 & 1-\lambda \end{bmatrix} = -\lambda^3 + 7\lambda^2 - 14\lambda + 8$$

With roots λ=1,λ=2 and λ=4.The corresponding eigenvectors are

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

respectively. Therefore we have $P^{-1}AP = D$ where

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}. \quad P^{-1} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}. \quad and \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Eigenvalues , eigenvectors and diagonalization

We can equivalently write $A = PDP^{-1}$. Finding a square root of a diagonal matrix is easy:

$$D^{\frac{1}{2}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

If we now define $B = PD^{0.5}P^{-1}$ we clearly have $B^2 = PD^{0.5}P^{-1}PD^{0.5}P^{-1} = PDP^{-1} = A$

So the desired square root of A is

$B = PD^{0.5}P^{-1} =$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1-\sqrt{2} & 1+\sqrt{2} & 1 \\ -1+\sqrt{2}1 & 1-\sqrt{2} & 1 \end{bmatrix}$$

Finally we verify that we have compated B correctly by squaring it and double-checking that we really get A.

$$B^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1-\sqrt{2} & 1+\sqrt{2} & 1 \\ -1+\sqrt{2} & 1-\sqrt{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1-\sqrt{2} & 1+\sqrt{2} & 1 \\ -1+\sqrt{2} & 1-\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ -1 & 5 & 3 \\ 1 & -1 & 1 \end{bmatrix} = A$$

We not that the square root of a matrix is not unique .In fact D has 8 different square roots ,all of the form

$$\begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm\sqrt{2} & 0 \\ 0 & 0 & \pm\sqrt{2} \end{bmatrix}.$$

It follows that A has 8 different square roots as well. We leave it as an exercise to compute them all.

The same method can also be used to compute other powers of a matrix, for example a cube root.

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