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mavzusidagi Respublika ilmiy-texnik anjuman materiallari to‘plami

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Globallashuv sharoitida davlatimizni yanada barqaror va jadal sur’atlar bilan rivojlantirish bo‘yicha amalga oshirilayotgan islohotlar samarasini yaxshilash sohasidagi ilmiy-tadqiqot ishlariga alohida e’tibor qaratilgan. Zero iqtisodiyotning, ijtimoiy sohalarni qamrab olgan modernizatsiya jarayonlari, hayotning barcha sohalarini liberallashtirishni talab qilmoqda.

Ushbu ilmiy ma’ruza tezislari to‘plamida mamlakatimiz va xorijlik turli yo‘nalishlarda faoliyat olib borayotgan mutaxassislar, olimlar, professor-o‘qituvchilar, ilmiy tadqiqot institutlari va markazlarining ilmiy xodimlari, tadqiqotchilari, magistr va talabalarning ilmiy-tadqiqot ishlari natijalari mujassamlashgan.

Mazkur to‘plamga kiritilgan ma’ruza tezislarining mazmuni, undagi statistik ma’lumotlar va me’yoriy hujjatlarning to‘g‘riligi hamda tanqidiy fikr-mulohazalar, keltirilgan takliflarga mualliflarning o‘zlari mas’uldirlar.

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ON POSITIVE EIGENVALUES OF THE DISCRETE SCHRÖDINGER OPERATOR WITH A NON-LOCAL POTENTIAL

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Abstract. On the $d-$dimensional lattice $Z^d, d \geq 1$ the discrete Schrödinger operator $H_{\lambda\mu}$ with a non-local potential constructed via the Dirac delta function and shift operator considered. The dependence of positive eigenvalues existence of the operator on the parameters is explicitly derived.

The discrete Schrödinger operator in the position representation. The standard discrete Laplacian Δ on the lattice Z^d, $d \geq 1$, is defined with the following self-adjoint (bounded) multidimensional Toeplitz-type operator on the Hilbert space $\ell^2(Z^d)$ ([3])

$$-\Delta = \frac{1}{2} \sum_{s \in Z^d |s|=1} (T(0) - T(s)),$$

where $T(y), y \in Z^d$ is the shift operator

$$(T(y)f)(x) = f(x+y), \quad f \in \ell^2(Z^d).$$

Let V_0 be a multiplication operator in $\ell^2(Z^d)$ by the Kronecker delta function $\delta[\cdot,0]$

$$V_0 f(x) = \delta[x,0] f(x).$$

Then, for a given point $x_0 \in Z^d$, we define the potential [3] as

$$V_{x_0} = \mu V_0 + \lambda (V_0 T(x_0) + T^*(x_0) V_0).$$

The discrete Schrödinger operator $\hat{H}_{\lambda\mu}$ acting in $\ell^2(\mathbb{Z}^d)$, in the position representation, is defined as a bounded self-adjoint perturbation of $-\Delta$ and is of the form $\hat{H}_{\lambda\mu} = -\Delta - \hat{V}$.

The discrete Schrödinger operator in momentum representation. The one-particle Hamiltonian $H_{\lambda\mu}$ in the momentum representation is given as

$$H_{\lambda\mu} = H_0 - V,$$

where H_0 is defined as

$$H_0 = \mathcal{F}^*(-\Delta)\mathcal{F},$$

where $\mathcal{F}$ stands for the standard Fourier transform $\mathcal{F} : L^2(\mathbb{T}^d) \to \ell^2(\mathbb{Z}^d)$ with the inverse $\mathcal{F}^* : \ell^2(\mathbb{Z}^d) \to L^2(\mathbb{T}^d)$. Explicitly, the non-perturbed operator H_0 acts on $L^2(\mathbb{T}^d)$ as a multiplication operator by the function $\varepsilon(\cdot)$

$$(H_0 f)(p) = \varepsilon(p) f(p), \ f \in L^2(\mathbb{T}^d),$$

where

$$\varepsilon(p) = \sum_{j=1}^{d} (1 - \cos p_j), \quad p \in \mathbb{T}^d.$$

In the physical literature, the function $\varepsilon(\cdot)$ being a real valued-function on $\mathbb{T}^d$, is called the *dispersion relation* of the Laplace operator.

The perturbation V is the two-dimensional integral operator

$$(Vf)(p) = (V_{x_0} f)(p) = \frac{1}{(2\pi)^d} \int_{\mathbb{T}^d} \left(\mu + \lambda(e^{i(x_0,p)} + e^{-i(x_0,s)})\right) f(s)ds, \ f \in L^2(\mathbb{T}^d).$$

The essential spectrum. The perturbation V of H_0 is a two dimensional operator, therefore in accordance with the Weyl theorem on the stability of the essential spectrum the equality $\sigma_{\text{ess}}(H_{\lambda\mu}) = \sigma_{\text{ess}}(H_0)$ holds, and the essential spectrum $\sigma_{\text{ess}}(H_{\lambda\mu})$ of the operator $H_{\lambda\mu}$ fills in the following interval on the real axis

$$\sigma_{\text{ess}}(H_{\lambda\mu}) = [\varepsilon_{\min}, \varepsilon_{\max}],$$

where

$$\varepsilon_{\min} = 0, \quad \varepsilon_{\max} = 2d.$$

The Fredholm determinant of $H_{\lambda\mu}$**.** Set

$$a(z)=\frac{1}{(2\pi)^d}\int_{\mathbb{T}^d}\frac{1}{e(t)-z}dt,\quad b(z)=\frac{1}{(2\pi)^d}\int_{\mathbb{T}^d}\frac{e^{i(x_0,t)}}{e(t)-z}dt,\quad z\in\mathbb{R}\setminus[e_{\min},e_{\max}].$$

For any $\lambda,\mu\in\mathbb{R},$ the Fredholm determinant associated to the operator $H_{\lambda\mu}$ as a regular function in $z\in\mathbb{C}\setminus[e_{\min},e_{\max}]$ is defined as

$$\Delta(\lambda,\mu;z)=(1-\lambda b(z))^2-\lambda^2a^2(z)-\mu a(z).$$

$$c(z)=a^2(z)-b^2(z).$$

For any $z\in\mathbb{R}\setminus[e_{\min},e_{\max}]$, the number $a(z)$ is non-zero, and the equality

$$\Delta(\lambda,\mu;z)=a(z)\left(\frac{1}{a(z)}-\frac{2b(z)}{a(z)}\lambda-\frac{c(z)}{a(z)}\lambda^2-\mu\right)$$

allows us to consider the parabola

$$P_z(\lambda,\mu):=\frac{1}{a(z)}-\frac{2b(z)}{a(z)}\lambda-\frac{c(z)}{a(z)}\lambda^2-\mu=0$$

in the $\lambda\mu$-plane with λ *-intercepts*

$$\kappa(z)=\frac{1}{b(z)+a(z)}\quad \textit{and}\quad \nu(z)=\frac{1}{b(z)-a(z)}$$

instead of the equality $\Delta(\lambda,\mu;z)=0$.

Lemma 1. *The number* $z\in\mathbb{C}\setminus[e_{\min},e_{\max}]$ *is an eigenvalue of* $H_{\lambda\mu}$ *if and only if* $\Delta(\lambda,\mu;z)=0$.

Lemma 2. *The parabola* $P_z(\lambda,\mu)=0$ *can be defined at* $e_{\max}$ *as*

$$P_{e_{\max}}(\lambda,\mu):=-2\lambda+\frac{2}{\kappa_r}\lambda^2-\mu=0\quad for\quad d=1,2\ \ with\ |x_0|_1-even,$$

$$P_{e_{\max}}(\lambda,\mu):=2\lambda-\frac{2}{\nu_r}\lambda^2-\mu=0\quad for\quad d=1,2\ \ with\ |x_0|_1-odd,$$

$$P_{e_{\max}}(\lambda,\mu):=\frac{1}{a_r}-\frac{2b_r}{a_r}\lambda-\frac{c_r}{a_r}\lambda^2-\mu=0\quad for\quad d\ge 3,$$

where

$$a_r := \lim_{z \to e_{\max}+} a(z),\ b_r := \lim_{z \to e_{\max}+} b(z),\ c_r := a_r^2 - b_r^2.$$

$$\kappa_r = \lim_{z \to 2d+} \kappa_r(z),\ \ \nu_r = \lim_{z \to 2d+} \nu_r(z)$$

Let Γ_r be graph of the parabola $P_{e_{\max}}(\lambda,\mu)=0$ in the $\lambda\mu$–plane. It divides the (λ,μ) plane into two disjoint parts

$$G_0^r = \{(\lambda,\mu): P_{e_{\max}}(\lambda,\mu) > 0\},\ G_1^r = \{(\lambda,\mu): P_{e_{\max}}(\lambda,\mu) < 0\}.$$

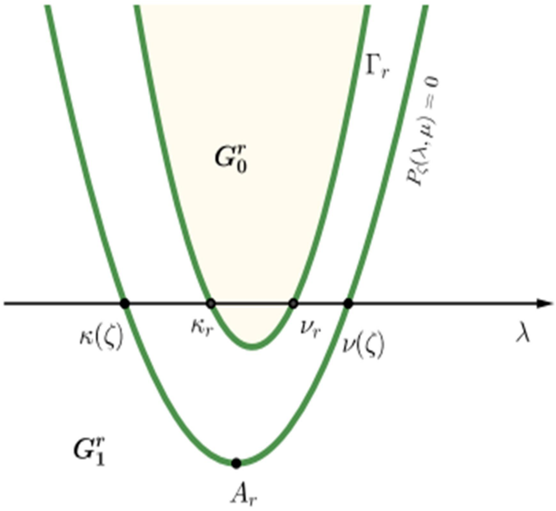

Figure : Location of $P_\zeta(\lambda,\mu)=0$, $\zeta > e_{\max}$.

Theorem 1.

(a) For $(\lambda,\mu) \in G_0^r \cup \Gamma_r$, *the operator* $H_{\lambda\mu}$ *has no eigenvalues in* $[e_{\max},\infty)$.

(b) For $(\lambda,\mu) \in G_1^r$, *the operator* $H_{\lambda\mu}$ *has a simple eigenvalue in* $[e_{\max},\infty)$.

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