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Globallashuv sharoitida davlatimizni yanada barqaror va jadal sur’atlar bilan rivojlantirish bo‘yicha amalga oshirilayotgan islohotlar samarasini yaxshilash sohasidagi ilmiy-tadqiqot ishlariga alohida e’tibor qaratilgan. Zero iqtisodiyotning, ijtimoiy sohalarni qamrab olgan modernizatsiya jarayonlari, hayotning barcha sohalarini liberallashtirishni talab qilmoqda.

Ushbu ilmiy ma’ruza tezislari to‘plamida mamlakatimiz va xorijlik turli yo‘nalishlarda faoliyat olib borayotgan mutaxassislar, olimlar, professor-o‘qituvchilar, ilmiy tadqiqot institutlari va markazlarining ilmiy xodimlari, tadqiqotchilari, magistr va talabalarning ilmiy-tadqiqot ishlari natijalari mujassamlashgan.

Mazkur to‘plamga kiritilgan ma’ruza tezislarining mazmuni, undagi statistik ma’lumotlar va me’yoriy hujjatlarning to‘g‘riligi hamda tanqidiy fikr-mulohazalar, keltirilgan takliflarga mualliflarning o‘zlari mas’uldirlar.

1-SHO'BA.
ANIQ VA TABIIY FANLARNING NAZARIY ASOSLARI, HISOBLASH VA EKSPERIMENTAL TADQIQOTLAR.

DISCRETE SPECTRUM OF TWO-FERMION SCHRÖDINGER OPERATORS ON 3D LATTICES

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Abstract

This thesis investigates the spectral properties of two-fermion Schrödinger operators on the three-dimensional lattice $\mathbb{Z}^3$ with nearest- and next-nearest-neighbor interactions. We focus on the discrete eigenvalues lying outside the essential spectrum and analyze their dependence on interaction parameters. The study employs decomposition into invariant subspaces and Fredholm determinant techniques to characterize bound states and critical phenomena.

Introduction

Lattice Schrödinger operators are fundamental in quantum physics for modeling interacting particles on discrete structures. Two-fermion systems are particularly interesting as they capture essential aspects of fermionic symmetry and bound state formation. Unlike continuous-space models, lattice systems exhibit discrete translational invariance, preventing full separation of the center-of-mass motion but allowing decomposition into fiber operators via quasi-momentum $K \in \mathbb{T}^3$.

In this thesis, we consider the Hamiltonian

$$H_{\lambda\mu}(K) = H_0(K) + V_{\lambda\mu}, \lambda, \mu \in R,$$

acting on the subspace $L^{2,o}(\mathbb{T}^3)$ of odd functions on the torus. Here, $H_0(K)$ is the free part given by

$$\varepsilon_K(p) = \varepsilon\left(\frac{K}{2}+p\right) + \varepsilon\left(\frac{K}{2}-p\right),\ \ \varepsilon(p) = \sum_{i=1}^{3}(1-\cos p_i)$$

and $V_{\lambda\mu}$ represents finite-range interactions. Our main goal is to determine the discrete spectrum of $H_{\lambda\mu}(K)$, particularly at $K = 0$, and to establish bounds on eigenvalues for general K.

Preliminaries

The operator $H_{\lambda\mu}(K)$ is bounded and self-adjoint. By Weyl's theorem, the essential spectrum coincides with that of $H_0(K)$:

$$\sigma_{ess}\left(H_{\lambda\mu}(K)\right) = \sigma\left(H_0(K)\right) = [E_{min}(K), E_{max}(K)],$$

where

$$E_{min}(K) = 4\sum_{i}^{3}\left(1-\cos\frac{K_i}{2}\right),\quad E_{min}(K) = 4\sum_{i}^{3}\left(1+\cos\frac{K_i}{2}\right)$$

The space $L^{2,o}(\mathbb{T}^3)$ decomposes into invariant subspaces labeled by parity:

$$L^{2,o}(\mathbb{T}^3) = \bigoplus_{\theta\in\{eeo,eoe,oee,ooo\}} L^{2,\theta}(\mathbb{T}^3)$$

with further decomposition into symmetric and antisymmetric parts under transpositions σ_θ. The operator $V_{\lambda\mu}$ vanishes on $L^{2,ooo}(\mathbb{T}^3)$, so analysis focuses on the remaining subspaces.

Discrete Spectrum at $K = 0$

Let $H_\mu^{\theta,asym}(0)$ denote the restriction of $H_{\lambda\mu}(0)$ to $L^{2,\theta,asym}(\mathbb{T}^3)$. These operators are unitarily equivalent for $\theta \in \{\mathrm{eeo}, \mathrm{eoe}, \mathrm{oee}\}$, allowing spectral analysis to focus on $\theta = eeo$. Discrete eigenvalues satisfy

$$H_\mu^{\theta,asym}(0)f = zf, \qquad z \notin [0,24].$$

Using the Fredholm determinant $\Delta_\mu(z)$,

$$\Delta_\mu(z) = 1 + \mu a(z), \qquad a(z) = \frac{1}{4\pi^3}\int_{T^3} \frac{\sin^2 p_1 (\cos p_3 - \cos p_2)^2}{\varepsilon_0(p) - z}, \qquad (8)$$

a number $z \in R\setminus [0,24]$ is an eigenvalue if and only if $\Delta_\mu(z) = 0$. There is at most one root in $(-\infty, 0)$ and one in $(24, +\infty)$, corresponding to bound states below or above the essential spectrum.

Main Results

- For $\mu \in [-\frac{1}{a(0)}, \frac{1}{a(0)}]$, the operator $H_\mu^{\theta,asym}(0)$ has no eigenvalues outside $[0,24]$.
- For $\mu > \frac{1}{a(0)}$, there exists exactly one discrete eigenvalue above 24.
- For $\mu < -\frac{1}{a(0)}$, there exists exactly one discrete eigenvalue below 0.
- These results extend to $H_{\lambda\mu}(K)$ for arbitrary $K \in T^3$, providing lower bounds on the number of discrete eigenvalues.

These findings reveal the emergence and disappearance of bound states induced by small perturbations in the interaction, defining critical points along the parameter μ.

Conclusion

The study demonstrates that the discrete spectrum of two-fermion Schro¨dinger operators on Z^3 is fully characterized by the Fredholm determinant of the antisymmetric subspace restriction. Critical values of μ determine the creation or annihilation of bound states at spectral edges. The framework provides a rigorous basis for analyzing stability and bifurcation of eigenvalues in lattice quantum systems, offering insights relevant for both theoretical and computational studies of discrete fermionic systems.

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KOSSERA MUHITIDA HARAKATLANUVCHI YUKDAN HOSIL BO'LADIGAN DINAMIK JARAYONLAR XUSUSIYATLARI

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Annotatsiya. Ushbu ishda harakatlanuvchi yuk ta'sirida Kossera muhitida hosil bo'ladigan kuchlanish-deformatsiya holati masalasi o'rganilgan. Qovushqoq-elastik yarim tekislikda to'lqin tarqalishi masalalarini yechish metodikasi va algoritmi ishlab chiqilgan, shuningdek analitik va sonli hisoblash usullari keltirilgan.

Kalit so'zlar: Kossera muhiti, kuchlanish-deformatsiya holati, yarim tekislik, qovushqoq-elastiklik, to'lqin tarqalishi.

1. Kirish.

Klassik elastiklik nazariyasi modeli ideallashtirilgan bo'lib, materialni zarralardan tuzilgan deb qaraydi va zarralar orasida yuzaga keladigan o'zaro ta'sir kuchlarini to'liq hisobga olmaydi [1,2]. Tajriba natijalari bilan nazariy hisoblashlar o'rtasidagi nomuvofiqlik, ayniqsa, yuqori chastotali (yoki qisqa to'lqin uzunlikdagi) to'lqinlarni tadqiq etishda kuzatiladi. Klassik elastiklik nazariyasi doirasida to'lqinlarning tarqalishida dispersiya hodisasini hamda antiplastik (yoki antiplanar) sirt to'lqinlarining mavjudligini to'liq tushuntirib bo'lmaydi [3–5]. Mazkur holat eksperimental jihatdan tasdiqlangan bo'lsa-da, hozirgacha yetarli darajada o'rganilmagan.

2. Masalaning qo'yilishi va yechish metodikasi

Faraz qilaylik, yarim tekislik sirtida nuqtaviy kuch Dtezlik bilan harakatlansin. Uning tezligi bo'ylama va ko'ndalang to'lqinlar tezligidan oshmaydi. Bu holda muhitda to'lqin tarqalishini ifodalovchi tenglamalar quyidagi ko'rinishga ega bo'ladi:

$$\frac{\partial \sigma_{x'x'}}{\partial x'}+\frac{\partial \sigma_{y'x'}}{\partial y'}=\rho\frac{\partial^2 u}{\partial t^2},\frac{\partial \sigma_{x'y'}}{\partial x'}+\frac{\partial \sigma_{y'y'}}{\partial y'}=\rho\frac{\partial^2 \vartheta}{\partial t^2}. \tag{1}$$

Kuchlanish komponentlari ham quyidagi ko'rinishda bo'ladi: